International Journal of Engineering Research & Management Technology

ISSN: 2348-4039

Volume 9, Issue-6 November-December- 2022

Email: editor@ijermt.org www.ijermt.org

"An Analytic Study on the Functions Differentiability In Terms Of Continuity under Selected Topological Structure"

Shaista Parveen Shaikh Mohammed[1]

[1]Research scholar, Department of Mathematics, Sunrise University Alwar, (Raj)

shaistaparveen2212@gmail.com

Dr. Manjeet Singh[2]

[2]Associate Professor, Department of Mathematics, Sunrise University Alwar, (Raj)

Abstract

Differential transform has the advantage of directly giving the solution of differential equation with the given boundary values without the necessity of first finding the general solution and then transform any real or complex variable bearing function into the integral form. In case of more than two and three variables so as for thermo dynamical phenomenon some variables are kept constant and partial differentiation comes into play. Wave equation, heat equation etc. are such equation which may not be solved with the help of Differential operator L. So these types of problem on which Differential operator or transform fails will be non-moment problem function. A Non-Moment function is directly proportional to any exponential function. The name Differential transform is derived from the French mathematician and astronomer Pierre-Simon Differential, who used similar type of transform theory in his work on probability theory. The transform theory was widespread at the time after world war 2nd. And in 19th century it is used by Abel, lerch, Heaviside and Bronwich.

Keyword: - Differential, phenomenon, proportional, probability.

INTRODUCTION

Suppose f is a real valued function defined on a subset D of R. We are going to define *limit* of f (x) as x $\in$ D *approaches* a point which is not necessarily in D.

First we have to be clear about what we mean by the statement "x $\in$ D *approaches* a point a".

Limit point of a set $D \subseteq$ R

Definition 1.2 Let D $\subseteq$ R and a $\in$ R. Then a is said to be a **limit point** of D if for any $\delta > 0$, the interval $(a-\delta, a+\delta)$ contains at least one point from D other than possibly a, i.e.,

$$D \cap \{x \in R : 0 < |x - a| < \delta\} = \varnothing.$$

Lemma 1.2 The statements in the following can be easily verified:

(i) Every point in an interval is its limit point.

(ii) If I is an open interval of finite length, then both the end points of I are limit points of I.

(iii) The set of all limit points of an interval I of finite length consists of points from I together with its endpoints.

(iv) If $D = \{x \in R : 0 < |x| < 1\}$, then every point in the interval $[-1, 1]$ is a limit point of D.

(v) If $D = (0, 1) \cup \{2\}$, then 2 is not a limit point of D. The set of all limit points of D is the closed interval $[0, 1]$

(vi) If $D = \{\frac{1}{n} : n \in N\}$, then 0 is the only limit point of D.

(vii) If $D = \{n/(n + 1) : n \in N\}$, then 1 is the only limit point of D.

For the later use, we introduce the following definition.

Definition 2.2 (i) For $a \in R$, an open interval of the form $(a-\delta, a+\delta)$ for some $\delta > 0$ is called a **neighbor-hood** of a; it is also called a **δ-neighbor-hood** of a.

(ii) By a **deleted neighbor-hood** of a point $a \in R$ we mean a set of the form $D_\delta := \{x \in R : 0 < |x-a| < \delta\}$ for some $\delta > 0$, i.e., the set $(a-\delta, a+\delta) \setminus \{a\}$.

With the terminologies in the above definition, we can state the following:
A point $a \in R$ is a limit point of $D \subseteq R$ if and only if every deleted neighbor-hood of a contains at least one point of D.

In particular, if D contains either a deleted neighbor-hood of a or if D contains an open interval with one of its end points is a, then a is a limit point of D.

Now we give a characterization of limit points in terms of convergence of sequences.

Theorem 2.1 *A point* $a \in R$ *is a limit point of* $D \subseteq R$ *if and only if there exists a sequence* (a_n) *in* $D \setminus \{a\}$ *such that* $a_n \to a$ *as* $n \to \infty$.

Proof. Suppose $a \in R$ is a limit point of D. Then for each $n \in N$, there exists $a_n \in D \setminus \{a\}$ such that

$a_n \in (a - 1/n, a + 1/n)$. Note that that $a_n \to a$.

Conversely, suppose that there exists a sequence (a_n) in $D\backslash\{a\}$ such that $a_n \to a$. Hence, for every $\delta > 0$, there exists $N \in \mathbb{N}$ such that $a_n \in (a-\delta, a+\delta)$ for all $n \geq N$. In particular, for $n \geq N$ $a_n \in (a-\delta, a+\delta) \cap (D \backslash \{a\})$.

Limit of a function *f* (*x*) as *x* approaches *a*

Deftnition 2.3 Let f be a real valued function defined on a set $D \subseteq R$, and let $a \in R$ be a limit point of D. We say that $b \in R$ is a **limit of** f(x) **as** x **approaches** a if for every $\varepsilon > 0$, there exists $\delta > 0$ such that

$|f(x) - b| < \varepsilon$ whenever $x \in D, 0 < |x-a| < \delta$, and in that case we write

$$\lim_{x \to a} f(x) = b$$

or

$$f(x) \to b \quad \text{as} \quad x \to a.$$

The relations in (*) in the above examples can also be written as

$$x \in D, \quad 0 < |x-a| < \delta \quad \Longrightarrow \quad |f(x) - b| < \varepsilon.$$

CONVENTION: In the following, whenever we talk about limit of a function f as x approaches $a \in R$, we assume that f is defined on a set $D \subseteq R$ and a is a limit point of D.

Also, when we talk about f (x), we assume that x belongs to the domain of f . For example, if we say that "f (x) has certain property P for every x in an interval I", what we mean actualy is that "f (x) has the property P for all $x \in I \cap D$, where D is he domain of f ".

Let D be an interval and a is either in D or a is an end point of D.

(i) Let f (x) = x. Since

$$|f(x) - a| = |x - a| \qquad \forall x \in D,$$

it follows that for any $\varepsilon > 0$, $|f(x) - a| < \varepsilon$ whenever $0 < |x - a| < \delta := \varepsilon$. Hence,

$$\lim_{x \to a} f(x) = a.$$

Let $f(x) = x^2$ and $\varepsilon > 0$ be given.

We show that $\lim_{x \to a} f(x) = a^2$.

$$|f(x) - a^2| = (|x| + |a|)|x - a| \qquad \forall\, x \in D,\ x \neq a.$$

Since $|x| \leq |x - a| + |a| \leq 1 + |a|$ whenever $|x - a| < 1$, we have

$$|f(x) - a^2| = (1 + 2|a|)|x - a| \qquad \forall\, x \in D,\ 0 < |x - a| \leq 1.$$

The Laplace transform is one of the most widely used transform having many applications in physics and engineering. It is denoted by L{f(t)}, it is a linear operator of a function f(t) with a real argument t(t≥0) that transforms it to a function f(s) with a complex arguments. This transformation is essentially injective for the majority of practical uses. The two respective pairs of f(t) and f(s) are seen in tables Laplace transformation is profitable which mean in that many relationships and operation over the images can be correspond to simpler relationships and operations over the images. Laplace transform is derived from the value of scientist Pierre-Simon Laplace who was work on his probability theory.

The Laplace transform is also related to the Fourier transform, but the Fourier transform tells about a function or signal as a series of modes of vibrations, where as the Laplace transform tells about a function into its moments.

Like the Fourier transform, the Laplace transform is used for solve the differential equation and integral equations. In physics and engineering the Laplace transform is used for analysis of linear time invariant system such as electrical circuits, harmonic oscillator's optical devices and mechanic system.

LITERATURE REVIEW

The reason of literature review is to formally assist familiarity with current thinking & research on a particular topic. Several studies have been conducted to explain moment problem related to Laplace transform, highlights the researcher's work related to the topic & some of the authors along with their research works are mentioned below –

Helie (2015)– Stablished the report lays emphasis on numerical optimization of physical models with realistic damping under the guidance & support of Consonnes. Under this observation optimization has not been yet completely acknowledged. It provided the serious drawback for real-time sound synthesis i.e. lack of time stability & less availability of convergence rates than hyperbolic counter.

Z latescu (2016) – reported on 2 dimensional complex L-moment problem on closed unit disc with sequence of data and secondary L-moment problem on arbitrary compact set. Problem with the solution of the Dirichlet problem for bi-harmonic equation in half plane

Bertsimas (2017) – Revised problem involving moment random variables & large number of related areas of mathematical economics research & providing a control role to play in moment problem arising in probability

D-Matiynon (2017) – Presented problem solving the numerical inversion of the Laplace transform in physical models already observed. It seems to be a minimal distance to singularities that the contour cannot go beyond without degenerating the scheme.

Ivic (2018) – Studied on applications of Laplace transforms to analytic number theory including the classical circle & divisor problem with the discussion on functional equations. Also find the generalization of Fourier-Laplace transform in the distributional sense which is useful to solve differential & integral equations.

METHODOLOGY

Let $I \subseteq R$ be an interval. For a positive measure μ on *I*the nth moment is defined as $\int_I x^n d\mu(x)$ provided the integral exists. If we suppose that $(s_n)_{n>=0}$ is a sequence of real numbers, the moment problem on *I* consists of solving the following three problems:

(I) There exist a positive measure on *I* with moment $(s_n)_{n>=0}$.

(II) This positive measure uniquely determined by the moments $(s_n)_{n>=0.}$

When μ is a positive measure with moments $(s_n)_{n>=0}$ we say that μ is a solution to the moment problem. If the solution to the moment problem is unique, the moment problem is called determinate otherwise the moment problem is said to be indeterminate.

(III) All positive measure on I with moments $(s_n)_{n>=0}$can we uniquely describe by the given historical reason ie. The moment problem on [0,1) is referred to as the Hausdroff's moment problem and the moment problem on R is called the Hamburger moment problem and thus [0,∞) is called the Stieltjes moment problem.

STATEMENT OF THE PROBLEM

Hausdroff``s moment problem often known as moment problem is given as below-

$$\{\mu_n\}_0^\infty : \mu_0, \mu_1, \mu_2, \ldots;$$

There are other forms of existence to predict function $\alpha(t)$ of bounded variation in interval (0,1) for example –

$$\mu_n = \int_0^1 t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots)$$

Hence, this is corollary known as moment sequence. But, it has been noticed that every sequence (3.1,1) has them form (3.1,2) since (3.1,2) implies that-

$$|\mu_n| \leqq V[\alpha(t)]_0^1$$

the quantity on the right being the variation of $\alpha(t)$ on the interval (0, 1). Thus, in this way its sequence is confined. It was F. Hausdorff [1921a] to determine essential situation of sequence to be moment sequence. But, given that representation (3.12) if $\alpha(t)$ is a normalized function of corollary differences.

$$\alpha(0)=0, \quad \alpha(t)=\frac{\alpha(t+)+\alpha(t-)}{2} \qquad (0 < t < 1).$$

As normalization of the function $\alpha(t)$ does not varies the value of the integral (3.1,2) so, we can take over the loss with any disturbance to general prospect $\alpha(t)$ which is normalized. Infact, the entire discussion throws light upon it without any obstacle.

STUDY OF THE SIGNIFICANCE

In order to obtain required and significantly an adequate conditions for the representation of function as a Laplace integral we require an introductory discussion of kernels of non-negative type. These are the continuous analogues of non-negative or semi-definite quadratic forms.

According to Definition A real function k(x,y) which is not discontinuous in the square ($\alpha < x < b$, $a < y < b$) is of non-negative type there if for every real function $\phi(x)$ continuous in ($\alpha < x < b$)

$$J(\phi) = \int_a^b \int_a^b k(x,y)\phi(x)\phi(y)\,dx\,dy \geqq 0$$

For instance, take k (x, y) = g(x) g (y) where g(x) is any function continuous in ($\alpha < x < b$).

Note:

The integral (7.20,86) may finish without having $\phi(x)$ identically zero. Thus is our example we have only to choose $\phi(x)$ orthogonal to g(x) on (a,b).

A kernel is said to be non-negative definite if it is non-negative type and if integral (7.20,86) can finish for no real continuous function $\phi(x)$ except $\phi(x) = 0$. As an example take a = 0, b = π and

$$k(x,y) = \sum_{n=0}^{\infty} e^{-n} \cos nx \cos ny.$$

The integral (7.20,86) becomes

$$\sum_{n=0}^{\infty} e^{-n} a_n^2$$

$$a_n = \int_0^{\pi} \phi(x) \cos nx \, dx \qquad (n = 0,1, 2,......).$$

But (7.20,87) cannot be zero until all the a_n are zero. But by the completeness of the cosine set on (0, π) this implies that $\phi(x)$ is identically zero.

To Proof: An important result of J. Mercer (1990) that brings out the connection between kernels and quadratic forms.

Axiom 7.20(a) A continuous kernel k (x,y) is of non-negative type if and only if for every non-infinite sequence $\{x_i\}_0^n$ of distinct number of (a < x < b) the quadratic form-

$$Q_n = \sum_{i=0}^{n} \sum_{i=0}^{n} k(x_i, x_i)\xi_i \xi_i$$

Is non-negative (definite or semi-definite).

CONCLUSION

This is such a partial differential equation whose solution finally contains some arbitrary constant and exponential function.

For the purpose of generating a new ideology and corollary on Non-Moment problem, we take into account equation as the standard form of an equation representing itself as the generator of non-moment problem function. To make our result more accurate and widely considerable, we would like to introduce some other facts and finding coherent to the convergence of result and making our formula as a very common men discussion.

In the Engineering Mechanics, “Moment” and in the basic function theory “exponential function” has been cause of concern for those who are working on Science and Technology. Though an exponential function generate an infinite series but consequently as far as number of terms in the series increases magnitude value of such particular term recedes.

Like is the situation of impact of applied forces recedes as far as it touches molecular part of the physical substance in a Moment is produced. It is not always true to advocate that that this force plays its role in the one dimension part of the said and taken substance for the purpose of testing the Moment generated so.

More than one variable are changed guiding the physical shape and molecular structure of the substance. Throughout the study made learned Mathematician Pierre de Laplace (1749-1827), Engineer Oliver Heaviside

(1850-1925), also by Bromwich and Carson during 1916-17. Here we can some basic treatment on an exponential series that leads to the concept of a Non-Moment problem.

REFERENCES

1. **Abate J. and W. Whitt. (2015).** Avoiding $M=G=1$ transform functional equations in numerical inversion in preparation.
2. **Agnihotri (2018).** The two dimensional transform concerning a common group of polynomials.
3. **Agrawal (2014).The formulation provides a means to compute the surface when given the temperature fluctuation.**
4. **Agrawal (2012)** Author established two theorems which have given the descriptions of the product of H-function and the common group of polynomials in Saigo operators.